EXPLORATION, continued

3. The symbol $f(g(x))$ is read "f of g of x." It means find the value of $g(x)$ first, and then find f of the result. For instance, $g(5) = 1.5$. So $f(g(5)) = f(1.5) = 5.5$. Put another column into the table for values of $f(g(x))$. Write "none" where appropriate.
4. Show in the table an instance where $g(x)$ is defined but $f(g(x))$ is not defined.
5. Plot the values of $f(g(x))$ on a copy of the figure in Problem 1. If the points do not lie in a straight line, go back and check your work.
6. The function in Problem 5 is called the *composition* of f with g, which can be written $f \circ g$. What are the domain and range of $f \circ g$?
7. Write equations for functions f and g.
8. Enter into your grapher the equations of f and g as $f_1(x)$ and $f_2(x)$, respectively. Use Boolean variables or enter the domain directly, depending on your grapher, to make the functions have the proper domains. Then plot the graphs. Does the result agree with the given figure?
9. Enter $f \circ g$ into $f_3(x)$ by entering $f_1(f_2(x))$. Plot this graph. Does it agree with the graph you drew in Problem 5?
10. By suitable algebraic operations on the equations in Problem 7, find an equation for $f(g(x))$. Simplify the equation as much as possible.
11. What did you learn as a result of doing this exploration that you did not know before?

Symbols for Composite Functions

Suppose that the radius of the ripple is increasing at the constant rate of 8 in./s. Then

$$r = 8t$$

where r is the radius in inches and t is the number of seconds. If $t = 5$, then

$$r = 8 \cdot 5 = 40 \text{ in.}$$

The area of the circular region is given by

$$a = \pi r^2$$

where a is the area in square inches and r is the radius in inches. At time $t = 5$, when the radius is 40, the area is given by

$$a = \pi \cdot 40^2 = 1600\pi = 5026.5482\ldots \approx 5027 \text{ in}^2 \text{ or about } 35 \text{ ft}^2.$$

The 5 s is the *input* for the radius function, and the 40 in. is the *output*. Figure 1-4b shows that the output of the radius function becomes the input for the area function. The output of the area function is 5026.5....

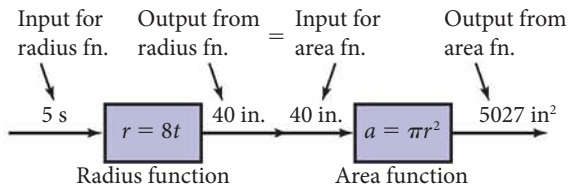

Figure 1-4b